FINAL EXAMINATION
December 7, 18:00-19:50 pm

OPEN BOOK - ALL PROBLEMS EQUALLY WEIGHTED

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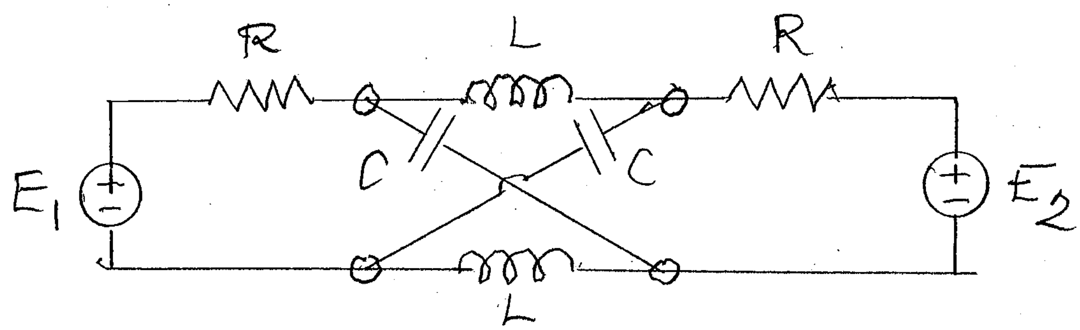
1. For the terminated two-port shown, L = 1 μH and C = 4 pF. The scattering parameter S11 is equal to 0 for all frequencies.

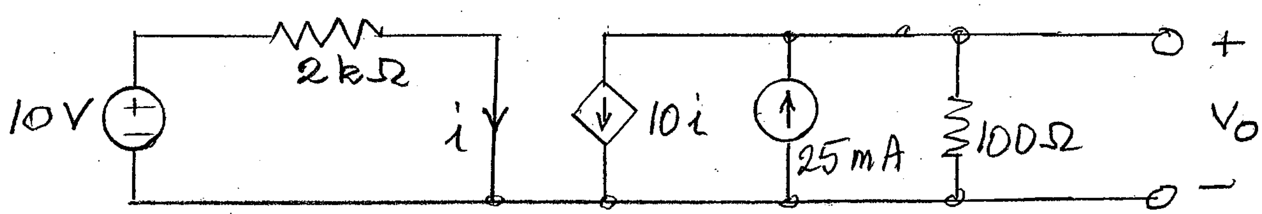
a. What are the impedance parameters zij of the twoport? What is the value of R?

b. What are the absolute values of the other three scattering parameters? Why?

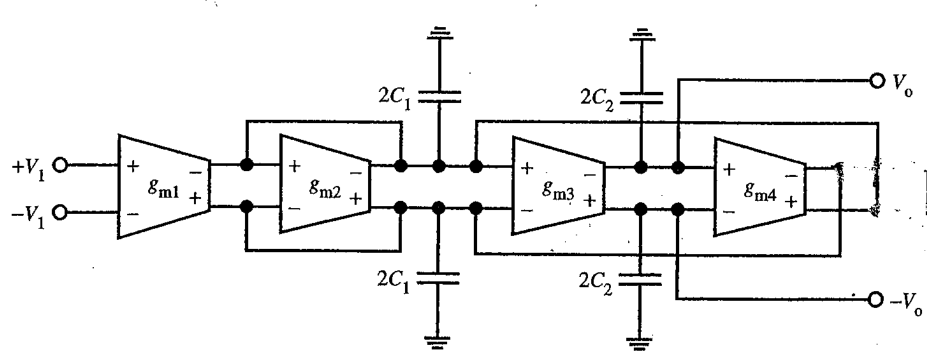
c. Derive S12(s) and S21(s)!



2. Find the Thevenin equivalent of the circuit shown using the adjoint network method.



3. Find the transfer function of the filter shown.



1.a.

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = -R I_2 = z_{21} I_1 + z_{22} I_2 \rightarrow I_2 = \frac{-z_{21} I_1}{z_{22} + R}$$

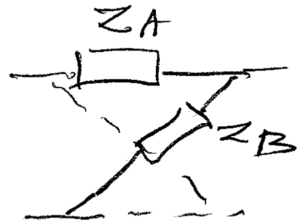
$$V_1 = I_1 \left[z_{11} - \frac{z_{12} z_{21}}{z_{22} + R} \right] = Z_{in} I_1$$

$$Z_{in} = \frac{z_{11}^2 - z_{12}^2 + R z_{11}}{z_{11} + R} \stackrel{!}{=} R$$

$$z_{11}^2 - z_{12}^2 = R^2 = (z_{11} + z_{12})(z_{11} - z_{12})$$

$$z_{11} = \frac{z_A + z_B}{2}, \quad z_{21} = z_{11} \frac{z_B - z_A}{z_B + z_A} = \frac{z_B - z_A}{2}$$

$$R^2 = z_A z_B = \frac{sL}{sC} = \frac{L}{C} \rightarrow R = \sqrt{\frac{L}{C}} = 500\,\Omega$$



$$z_{11} = \frac{s^2 LC + 1}{2sC} = z_{22}$$

$$z_{12} = \frac{1 - s^2 LC}{2sC} = z_{21}$$

$$z_{11}, z_{12} = \frac{1 \pm 4 \cdot 10^{-18} s^2}{8 \times 10^{-12} s}$$

b. By symmetry, $S_{22} \equiv 0$.
Since both ports are matched, and twoport is lossless, $|S_{12}| = |S_{21}| \equiv 1$.

1. c

$$z_{11} = z_{22} = \frac{z_A + z_B}{2}$$

$$z_{12} = z_{21} = \frac{z_B - z_A}{z_B + z_A} z_{11} = \frac{z_B - z_A}{2}$$

$$V_1 = \frac{E_1}{2} = z_{11}\frac{E_1}{2R} - z_{12}\frac{V_2}{R}$$

$$V_2 \frac{z_{12}}{\cancel{R}} = \frac{E_1}{2\cancel{R}}\left[z_{11} - R\right]$$

$$S_{21} = \frac{V_2}{E_1/2} = \frac{z_{11} - R}{z_{12}} = \frac{z_A + z_B - 2R}{z_B - z_A} = S_{12}$$

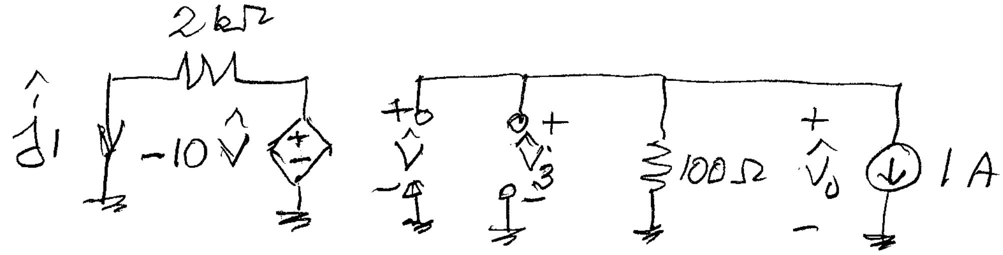
$$= \frac{sL + 1/sC - \sqrt{L/C}}{1/sC - sL} = \frac{s^2LC - 2s\sqrt{LC} + 1}{1 - s^2LC}$$

$$S_{\substack{12\\21}} = \frac{(s\sqrt{LC} - 1)^2}{(1 + s\sqrt{\ })(1 - s\sqrt{\ })} = \frac{1 - sT}{1 + sT} \qquad T \triangleq \sqrt{LC}$$

$$T = 2\,\text{ns}$$

$$|S_{21}(j\omega)| = \left|\frac{1 - j\omega T}{1 + j\omega T}\right| \equiv 1 \quad \forall\, \omega$$

2. The adjoint network is

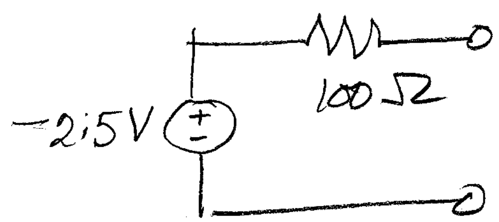


$\hat{V} = \hat{V}_3 = \hat{V}_0 = -100\ V$, $\hat{j}_1 = \dfrac{+10 \times 100}{2000} = +0.5\ A$

$V_0 = -\hat{j}_1 e + \hat{V}_3 i = -0.5 \times 10 + (-100)(-0.025) = -2.5V$

(From eq. 9-87)

$Z_{Th} = \hat{Z}_{Th} = -\hat{V}_0 = 100\,\Omega$



Easy to check from $\mathcal{N}$!

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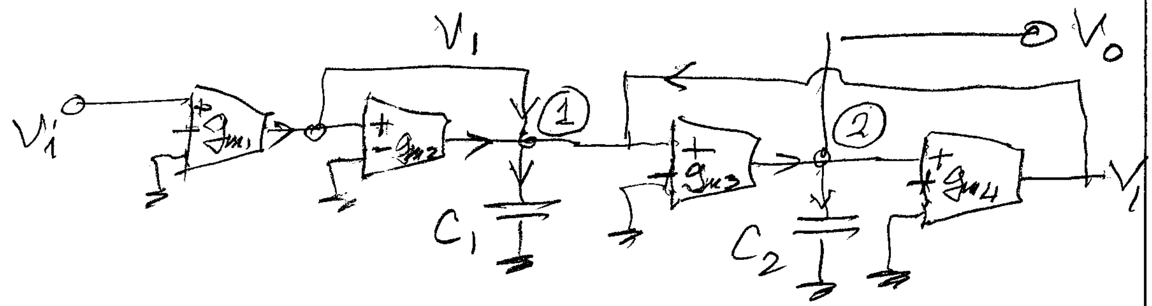
3.



KCL @ 1 :

$$-g_{m1}V_i - g_{m2}V_1 + g_{m4}V_o = sC_1 V$$

$$V_1 = (g_{m4}V_o - g_{m1}V_i)/(sC_1 + g_{m2})$$

KCL @ 2

$$-g_{m3}V_1 = sC_2 V_o$$

$$g_{m4}V_o - g_{m1}V_i = (sC_1 + g_{m2})\frac{sC_2}{-g_{m3}}V_o$$

$$[g_{m3}g_{m4} + (sC_1 + g_{m2})sC_2]V_o = g_{m1}g_{m3}V_i$$

$$\frac{V_o}{V_i} = g_{m1}g_{m3}/(s^2C_1C_2 + s\,g_{m2}C_2 + g_{m3}g_{m4})$$